

Platelet Collapse Model of Pulsar Glitches

P.D. Morley
XonTech, Inc.
6862 Hayvenhurst Ave.
Van Nuys, CA 91406

and

Iván Schmidt¹
Department of Physics
Universidad Técnica Federico Santa María
Casilla 110-V
Valparaíso, Chile

Abstract

A platelet collapse model of neutron starquakes is introduced. It displays self-organized criticality with a robust power-law behavior. The simulations indicate a near-constant exponent, whenever scaling is present.

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Bak, Tang and Wiesenfeld [1] introduced the concept of self-organized criticality (SOC): large dissipative dynamical systems tend to drive themselves to a critical state without the need to fine-tune system parameters. SOC seems to provide a natural explanation for the ubiquitous scale-free phenomena seen in nature. The critical state is a non-equilibrium state characterized by spatial correlation functions exhibiting power-law behavior. Until the finite size of the system comes into play, there is no intrinsic length scale in the dynamics, giving rise to the observed scaling. Thus the outstanding feature of SOC is the appearance of avalanches of dynamical activity of all sizes. Recently Bak [2] has given examples of SOC in astrophysics, including light emitted from quasars, emission of hard x-rays from solar flares and pulsar glitches. In this paper we present a detailed model of pulsar glitches, which realizes SOC, characterized by partial or full collapse of the neutron star's surface induced by crustal strain.

A pulsar glitch is an essentially instantaneous discontinuity of the rotating neutron star's period to shorter values. Morley and García-Pelayo [3] have taken the world's pulsar glitch data and have shown empirically that the probability $P(E)$ that a macro-glitch of energy E can occur in a glitching pulsar has the scaling law

$$P(E) \sim E^{-\gamma} \quad (1)$$

where the critical exponent, γ , due to the small number of recorded glitches, is not a well-determined number, lying in the range $+1 < \gamma < +2$.

There are many hundreds of known pulsars, of which only a minute number actually glitch. Morley [4] has shown that a reasonable explanation of why pulsars glitch is the following: some fraction of pulsars are embedded in nebula or in regions of high interstellar mass density which allows them to accrete matter. All neutron stars and white dwarfs have an inherent instability induced by mass accretion, due to their inability to hydrostatically change their configuration. As one of these stars accretes matter and goes to a higher mass, its radius is also increasing which pushes the star to radial instability². This mass accretion stress is added to the stress caused by the torque on the star, which causes the pulsar to be an ellipsoid of revolution for the wrong angular speed. Together, these stresses induce tremendous strain in the neutron star's surface crust. When the threshold level is exceeded, the

²Masses of neutron stars and white dwarfs in stable equilibrium are monotonic *decreasing* functions of their radius.

crust relaxes, moving inward, resulting in a starquake. By conservation of angular momentum, this inward movement of surface crust causes the pulsar to immediately speed-up, the pulsar glitch.

We now model this dynamical process. A pulsar has a misalignment of its spin axis \vec{S} , with its magnetic dipole axis \vec{M} (see fig. 1). In the \vec{M} frame of reference, Ruderman [5] has argued that the surface of the star is divided into platelets which have varying surface sizes, constrained by the requirement that the total magnetic flux threaded through a platelet is approximately constant. We adopt the Olami et al. [6] cellular automaton model of earthquakes to this starquake model. Each platelet referenced (i,j) is connected to neighbors by mean shear moduli μ_s . The displacement of each platelet from its relaxed position is defined as $dr_{i,j}$. The total radial stress on a given platelet is

$$S_{i,j} = \mu_s^{lo}[2dr_{i,j} - dr_{i-1,j} - dr_{i+1,j}] + \mu_s^{la}[2dr_{i,j} - dr_{i,j-1} - dr_{i,j+1}] + \mu_c dr_{i,j} . \quad (2)$$

In eq(2) μ_s^{lo}, μ_s^{la} and μ_c are respectively the longitudinal, latitudinal shear moduli and the compression modulus. The latter stress is associated with mass accretion, while the former are shear stresses. The total stress on a platelet increases uniformly (with a rate proportional to $\frac{dR}{dt}$, where R is the star's radius) until one platelet reaches its threshold value and relaxes. This triggers the starquake. The redistribution of stress after a platelet collapse at position (i,j) is

$$\begin{aligned} S_{i\pm 1,j} &\rightarrow S_{i\pm 1,j} + \Delta S_{i\pm 1,j} \\ S_{i,j\pm 1} &\rightarrow S_{i,j\pm 1} + \Delta S_{i,j\pm 1} \\ S_{i,j} &\rightarrow 0 \end{aligned} \quad (3)$$

where

$$\begin{aligned} \Delta S_{i\pm 1,j} &= \frac{\mu_s^{lo}}{2\mu_s^{lo} + 2\mu_s^{la} + \mu_c} S_{i,j} = \beta^{lo} S_{i,j} \\ \Delta S_{i,j\pm 1} &= \frac{\mu_s^{la}}{2\mu_s^{lo} + 2\mu_s^{la} + \mu_c} S_{i,j} = \beta^{la} S_{i,j} . \end{aligned} \quad (4)$$

For the purposes of this paper, we put $\beta^{lo} = \beta^{la} = \beta$. Eqs(2)-(4) constitute the Olami et al. cellular automaton model [6]. We now add the important changes intrinsic to starquakes. We designate $\lambda(\theta_i, \phi_j)$ as the surface platelet

density function; that is, the platelet whose center is at position (i,j) has an area $\lambda(\theta_i, \phi_j)$ in the \vec{M} -system. Since the radial magnetic field is $M_R = M_0 \cos(\theta_{\vec{M}})$ we parametrize $\lambda(\theta_i, \phi_j)$ as (θ_i, ϕ_j are \vec{M} system angles)

$$\lambda(\theta_i, \phi_j) = \frac{C}{a + \cos(\theta_i)^2} . \quad (5)$$

The constant C is determined by the area normalization condition. We adopt a spherically shaped pulsar, and account for its eccentricity stresses directly as seen below. To handle the boundary topology of a sphere, we disconnect the pole and anti-pole points from their neighbors. By discretizing the $\theta_{\vec{M}}$ variable into M values, C satisfies $C \sum_{i=2}^{M-1} \frac{\Delta x_i}{a+x_i^2} = 2(1 - \Delta x/(a+1))$, where $x = \cos(\theta_{\vec{M}})$, $\Delta x_i = \Delta x = 2/M$, $x_i = \frac{2i-M-1}{M-1}$. ϕ is discretized into N values by $\Delta\phi = \frac{2\pi}{N}$, so $\phi_j = j\Delta\phi$. There are thus $N \times (M-2)$ platelets in the \vec{M} -system covering the star. In eq(5), a is the parameter which determines the ratio of the largest to smallest platelet sizes $\simeq \frac{1}{a+a^2}$. Since the eccentricities (ϵ, ϵ_0) of pulsars are minute, we adopt a spherical configuration. To account for the centrifugal strain of having ϵ deformation instead of ϵ_0 (strain-free), we introduce a platelet threshold strain, $S_{th}(\theta_{\vec{S}})$, which is a function of $\sin(\theta_{\vec{S}})$, where $\theta_{\vec{S}}$ is the polar angle in the \vec{S} -system:

$$S_{th}(\theta_{\vec{S}})_{i,j} = S_T(1 - \alpha |\sin(\theta_{\vec{S}})|_{i,j}) . \quad (6)$$

α in eq(6) is the parameter which accounts for the reduced threshold for platelet collapse induced by the centrifugal strain. All stresses are in units of S_T , which can be put equal to 1. By geometry $|\sin(\theta_{\vec{S}})|_{i,j} = (1 - (\sin(\psi) \cos(\phi_j) \sqrt{1 - x_i^2} + x_i \cos(\psi))^2)^{1/2}$, where ψ is the angle between \vec{S} and \vec{M} . The last major change is the new boundary conditions. All platelets except those ringing the pole and anti-pole have continuous boundary conditions of four neighbors. Those ringing the poles have only three. The parameters of the model are ψ, a, α, β and the number $N \times (M-2)$. The simulation proceeds as follows:

- 1) Initialize all platelet stresses to a random value between 0 and $S_{th}(\theta_{\vec{S}})_{i,j}$.
- 2) If any $S_{i,j} \geq S_{th}(\theta_{\vec{S}})_{i,j}$, the platelet relaxes. Redistribute the stress on $S_{i,j}$ according to eq(3) and add the areas of the collapsed platelets.

- 3) Repeat step 2) until the starquake is fully evolved.
- 4) Locate the platelet with the largest strain such that $\Delta S = S_{th}(\theta_{\vec{g}})_{i,j} - S_{i,j}$ is smallest.
- 5) Add ΔS to all platelets (uniform mass accretion) and return to step 2).

The energy of a starquake comes from gravity. We assume the platelets fall a fixed relative distance $\frac{\Delta R}{R}$, so the energy is just proportional to the area of the collapsed platelets: $E \sim \sum_{collapsed\ i,j} \lambda(\theta_i, \phi_j)$. Interestingly enough, there are rare starquakes where more than the total surface area collapses due to the near-threshold level of all platelets and the continuous boundary conditions.

In fig. 2, we present $\log \frac{dN}{dE}$ versus $\log E$, with the listed parameters³. No SOC is present. This can be understood rather simply: the value of a means there is a disproportionate area size distribution among platelets which means that the starquakes are dominated by the large platelet collapses. By changing a to a value where the area density is not greatly varying, fig. 3, SOC is realized. Simulations indicate that SOC is present for all small a values and the exponent is almost independent of the values of ψ and α . Only variation in β produces a weak modification of the scaling exponent, fig. 4.

Our computer results strongly support SOC in the phenomena of pulsar glitches. They indicate that mass accretion is the critical element driving the neutron star instability. The critical exponent should lie between $1.5 < \gamma < 2$, depending only on the ratios of shear and compression moduli.

³All runs had 999999 starquakes.

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Figure Captions

1. Platelets on the rotating neutron star with misaligned spin \vec{S} and magnetic dipole \vec{M} axis.
2. $\log \frac{dN}{dE}$ versus $\log E$, for $\psi=0.5$, $a=0.01$, $\alpha=.1$, $N = M = 50$ and $\beta=0.2$.
3. Same as fig. 2, except $a=2.0$.
4. Critical exponent γ versus stress parameter β .

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